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LETTER TO THE EDITOR

The polar-type density of states in a non-unitary odd-parity superconducting state with a gap with point nodes

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Abstract
It is shown that a density of states (DOS) proportional to the excitation energy, a so-called polar-like DOS, can arise in odd-parity states, with the superconducting gap vanishing at points even though the spin–orbit interaction for Cooper pairing is strong. Such gap structures are realized in the non-unitary states, $F_{1u}(1, i, 0)$, $F_{1u}(1, \varepsilon, \varepsilon^2)$, and $F_{2u}(1, i, 0)$, classified by Volovik and Gorkov (1985 *Sov. Phys.–JETP* **61** 843). This is due to the gap vanishing in a quadratic manner around a point on the Fermi surface.

In the early stages of research into the heavy-fermion superconductors, it was important to infer the gap anisotropy from the power law of the temperature dependence of a series of physical quantities [1–4]. It was a sort of golden rule there that a point node(s) of the superconducting gap leads to the density of states (DOS) $N_s(\omega) \propto \omega^2$, while a line node(s) leads to $N_s(\omega) \propto \omega$. It was also emphasized that all the odd-parity pairings would have only point node(s) if the spin–orbit coupling for the pairing interaction were so strong that the spin and orbital degrees of freedom of the gap function could not change independently [5–9]. However, it is not so self-evident whether the spin–orbit coupling for pairing is really so strong as to technically quench the independent variations in spin and orbital space [10, 11]. In any case, the classification scheme proposed by Volovik and Gorkov (VG) has been believed to rule out a polar-like DOS for the odd-parity states. The purpose of this letter is to point out that three of the non-unitary states in the VG scheme have a polar-like DOS, because the k-dependence of the gap around the point node is quadratic rather than linear.

In the odd-parity manifold, the quasiparticle energy is a matrix in the representation of spin eigenstates, as follows [12]:

$$\hat{E}_k = [\xi_k^2 \hat{1} + \hat{\Delta}_k^\dagger \hat{\Delta}_k]^{1/2}, \tag{1}$$

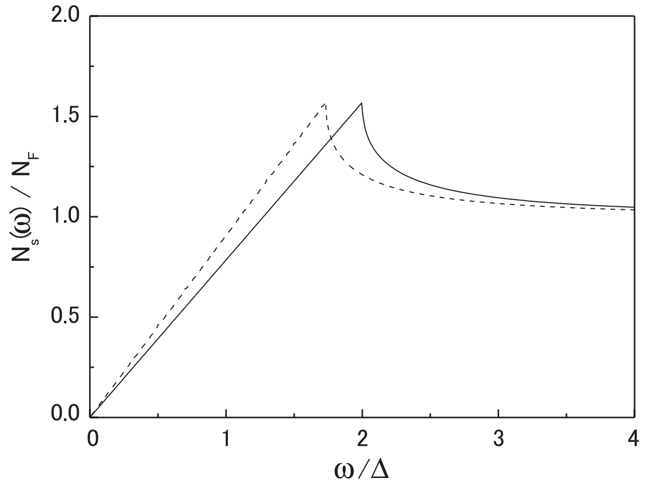

Figure 1. $N_{\rm s}(\omega)/N_{\rm F}$ in the states $\mathrm{F}_{1\mathrm{u}}(1, \mathrm{i}, 0)$ and $\mathrm{F}_{2\mathrm{u}}(1, \mathrm{i}, 0)$, $N_{\rm F}$ being the DOS in the normal state at the Fermi level. The dashed curve is for the DOS of the polar state $\Delta_{\boldsymbol{k}} = \sqrt{3}\Delta\hat{k}_z$.

where $\xi_{\boldsymbol{k}}$ is the energy of quasiparticles measured from the chemical potential; the superconducting gap is also a 2×2 matrix in the representation in spin eigenstates, and is represented in terms of the d-vector as

$$\hat{\Delta}_{\boldsymbol{k}} = \mathrm{i}\sum_j \Delta(\sigma_j\sigma_y)d_j(\boldsymbol{k}), \tag{2}$$

where σ_j is the Pauli matrix of the jth component, with $j = x$, y, and z. The eigenvalues of the magnitude of the gap matrix are given as [12]

$$(\hat{\Delta}^\dagger_{\boldsymbol{k}}\hat{\Delta}_{\boldsymbol{k}})_\pm = \Delta^2[(\boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{d}^*(\boldsymbol{k})) \pm |\boldsymbol{d}(\boldsymbol{k}) \times \boldsymbol{d}^*(\boldsymbol{k})|]. \tag{3}$$

It is remarked that the time reversal symmetry is broken in the non-unitary state where $\mathrm{i}\boldsymbol{d}(\boldsymbol{k}) \times \boldsymbol{d}^*(\boldsymbol{k}) \neq 0$. The DOS in the superconducting state $N_{\rm s}(\omega)$ is expressed as follows:

$$\frac{N_{\rm s}(\omega)}{N(0)} = \frac{\omega}{2}\sum_{\alpha=\pm}\int\frac{\mathrm{d}\hat{\boldsymbol{k}}}{4\pi}\,\frac{\theta(\omega - (\hat{\Delta}^\dagger_{\boldsymbol{k}}\hat{\Delta}_{\boldsymbol{k}})_\alpha)}{\sqrt{\omega^2 - (\hat{\Delta}^\dagger_{\boldsymbol{k}}\hat{\Delta}_{\boldsymbol{k}})_\alpha}}, \tag{4}$$

where $\theta(x)$ is the Heaviside function.

The d-vector of the state $\mathrm{F}_{1\mathrm{u}}(1, \mathrm{i}, 0)$, in the class of the group theoretical representation $D_4(E)$, is given as [6]

$$\boldsymbol{d}(\boldsymbol{k}) = \Delta(\tfrac{3}{4})^{1/2}[(\hat{k}_z\hat{\boldsymbol{e}}_y - \hat{k}_y\hat{\boldsymbol{e}}_z) + \mathrm{i}(\hat{k}_x\hat{\boldsymbol{e}}_z - \hat{k}_z\hat{\boldsymbol{e}}_x)], \tag{5}$$

where $\hat{\boldsymbol{k}} \equiv \boldsymbol{k}/|\boldsymbol{k}|$. Then, the magnitude of the gap is calculated, leading to the expression

$$(\hat{\Delta}^\dagger_{\boldsymbol{k}}\hat{\Delta}_{\boldsymbol{k}})_\pm = \tfrac{3}{4}\Delta^2(1 \pm |\hat{k}_z|)^2. \tag{6}$$

It is remarked that the amplitude of the smaller gap $[(\hat{\Delta}^\dagger_{\boldsymbol{k}}\hat{\Delta}_{\boldsymbol{k}})_-]^{1/2}$ has point nodes in the direction $|\hat{k}_z| = 1$, and has a quadratic dependence as $\propto(\hat{k}_x^2 + \hat{k}_y^2)$ around a node on the Fermi sphere. Therefore, the DOS is proportional to the excitation energy ω. With the use of (6), the DOS is calculated numerically by means of the formula (4). The result is shown in figure 1. The shape of the DOS is similar to those for polar states.

The d-vector of the state $\mathrm{F}_{2\mathrm{u}}(1, \mathrm{i}, 0)$, in the class of the group theoretical representation $D_4(E)$, is given as [6]

$$\boldsymbol{d}(\boldsymbol{k}) = \Delta(\tfrac{3}{4})^{1/2}[(\hat{k}_z\hat{\boldsymbol{e}}_y + \hat{k}_y\hat{\boldsymbol{e}}_z) + \mathrm{i}(\hat{k}_x\hat{\boldsymbol{e}}_z + \hat{k}_z\hat{\boldsymbol{e}}_x)]. \tag{7}$$

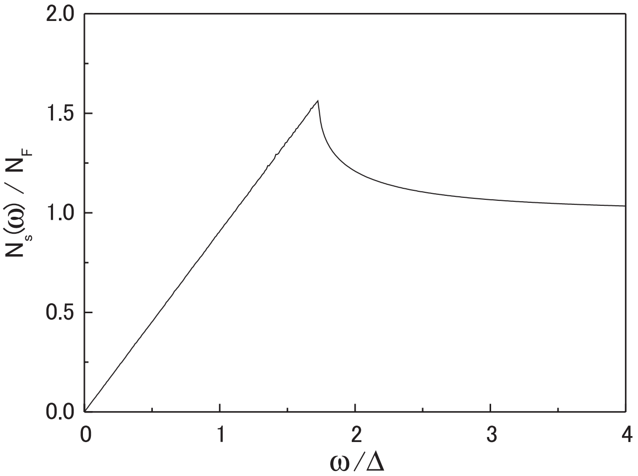

Figure 2. $N_{\mathrm{s}}(\omega)/N_{\mathrm{F}}$ in the state $\mathrm{F}_{1\mathrm{u}}(1, \varepsilon, \varepsilon^2)$, N_{F} being the DOS in the normal state at the Fermi level. This is the same as that of the polar state $\Delta_{\boldsymbol{k}} = \sqrt{3}\Delta\hat{k}_z$ within numerical errors.

The magnitude of the gap is calculated, leading to the same expression, equation (6). Therefore, the DOS $N_{\mathrm{s}}(\omega)$ is the same as that shown in figure 1, a polar-like one.

The d-vector of the state $\mathrm{F}_{1\mathrm{u}}(1, \varepsilon, \varepsilon^2)$, in the class of the group theoretical representation $D_3(E)$, is given as [6]

$$\boldsymbol{d}(\boldsymbol{k}) = \Delta(\tfrac{1}{2})^{1/2}[(\hat{k}_z\hat{\boldsymbol{e}}_y - \hat{k}_y\hat{\boldsymbol{e}}_z) + \varepsilon(\hat{k}_x\hat{\boldsymbol{e}}_z - \hat{k}_z\hat{\boldsymbol{e}}_x) + \varepsilon^2(\hat{k}_y\hat{\boldsymbol{e}}_x - \hat{k}_x\hat{\boldsymbol{e}}_y)], \tag{8}$$

where $\varepsilon \equiv \mathrm{e}^{\mathrm{i}2\pi/3}$. Then, the magnitude of the gap is calculated, leading to the expression

$$(\hat{\Delta}_{\boldsymbol{k}}^{\dagger}\hat{\Delta}_{\boldsymbol{k}})_{\pm} = \frac{\Delta^2}{4}(\sqrt{3} \pm |\hat{k}_x + \hat{k}_y + \hat{k}_z|)^2. \tag{9}$$

The smaller gap $[(\hat{\Delta}_{\boldsymbol{k}}^{\dagger}\hat{\Delta}_{\boldsymbol{k}})_{-}]^{1/2}$ has point nodes in the direction $\hat{\boldsymbol{k}} = (\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3})$, and also has a quadratic dependence as (6). So, the DOS is proportional to the excitation energy ω, and its explicit dependence is calculated numerically by means of the formula (4). The result is shown in figure 2. The DOS is the same as that for the polar state $\Delta_{\boldsymbol{k}} = \sqrt{3}\Delta\hat{k}_z$.

Another example in which the point node(s) gives a polar-like DOS is the so-called planar state with $\mathrm{E}_{2\mathrm{u}}$ symmetry which is a unitary state and was proposed as a candidate for being that of UPt_3 [13, 14]. Such a state gives the magnitude of the gap as

$$|\Delta_{\boldsymbol{k}}| \propto \hat{k}_z[(\hat{k}_x^2 - \hat{k}_y^2)^2 + 4\hat{k}_x^2\hat{k}_y^2]^{1/2}. \tag{10}$$

This gap has point nodes at $|\hat{k}_z| = 1$ and shows quadratic behaviour around a node on the Fermi surface. So, the quasiparticles around the point nodes should also give a polar-like DOS if there is a Fermi surface around the nodes.

In conclusion, we have indicated by means of explicit calculations that the superconducting gap with point nodes in the non-unitary states, $\mathrm{F}_{1\mathrm{u}}(1, \mathrm{i}, 0)$, $\mathrm{F}_{1\mathrm{u}}(1, \varepsilon, \varepsilon^2)$, and $\mathrm{F}_{2\mathrm{u}}(1, \mathrm{i}, 0)$, classified by VG, exhibits a polar-like DOS which is proportional to the excitation energy itself rather than its square. This result arises from the k-dependence of the gap around the point nodes being quadratic, rather than linear as expected in general.

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